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Procedia Engineering 111 (2015) 510 – 515

Procedia Engineering

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XXIV R-S-P seminar, Theoretical Foundation of Civil Engineering (24RSP) (TFoCE 2015)

Dual approach to solving the problems of structural optimization

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Abstract

Structural optimization problems are normally formulated and solved in terms of mathematical programming. Unknown parameters of structure must satisfy the constraints of the problem and provide extreme value of objective function (for instance minimum weight or maximum load-bearing capacity of considering structure). It is known that in some cases the optimal systems have special properties. Knowledge of these properties allows investigator to develop methods for solving optimization problems and methods for creating systems with prescribed properties.

Peer-review under responsibility of organizing committee of the XXIV R-S-P seminar, Theoretical Foundation of Civil Engineering (24RSP)

Keywords: Optimization; Structures with minimum materials consumption; Special properties; Safety of structures; Frequency; Bending mode; Buckling mode; Natural mode

1. Introduction

Two types of formulations of the problems (certainly with the same solutions) are normally considered in the optimization theory. The first one is formulated as a problem of finding a set of variable parameters which satisfy the constraints and provide minimum (maximum) value of the objective function. The second one is based on the identification of specific properties of optimal systems. It is formulated as a problem of the synthesis of structures with desired properties. This direction of research traces its roots from well-known Lagrange problem dealing with optimal shape of the column, loaded by axial force [6]. Solution of this problem was in area of expertise of T. Clausen [3]. In 1907, E.L. Nicolai [8] not only summarized the solutions of Lagrange and T. Clausen, but also formulated a special property of the bar of least weight by limiting the value of the critical (buckling) force. He proved that, under certain conditions, the bar of the minimum weight for a given value of the critical force will be

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Peer-review under responsibility of organizing committee of the XXIV R-S-P seminar, Theoretical Foundation of Civil Engineering (24RSP)
doi:10.1016/j.proeng.2015.07.124

bar of equal resistance with respect to the diagram of moments arising in the loss of stability (buckling). Later, N.G. Chentsova [12], A.F. Smirnov [11], A.I. Vinogradov [2], Ya.L. Nudelman [9], N. Olhoff [10] and others identified and presented in their works a lot of other special properties of optimal systems.

These properties depend on the class of structures, types of varied parameters and a set of constraints. For instance, under certain conditions beam with restrictions on strength and varied cross-sectional dimensions transforms to beam of equal strength. Special properties of natural modes are under consideration in cases of limiting the value of the first natural frequency or varied stiffness of discrete constraints [7]. There are some other properties of the systems of minimum material consumption.

The identified properties can be considered as a basis for the development of synthesis method. *The main aim is to find the minimum (maximum) value of objective functional (function) under given constraints. It is replaced by the problem of the synthesis of systems with known properties.* This formulation not only allows investigator to develop effective computational algorithms of optimization, but also to assess (with a high degree of reliability) the level of approximation of solutions obtained by conventional methods in comparison with optimal ones.

Some special properties of optimal systems and samples of their application (as a basis for development optimization algorithms) are presented below.

Problems of rational arrangement of constraints in problems of stability and natural vibrations have been considered by numerous scientists. Paper of I.G. Bubnov [1] was the pioneering and interesting for specialists in this field. We should also mention here, in particular, research works of Ya.L. Nudelman, A.F. Smirnov, M.D. Dolberg and others [4,5,9,11]. They demonstrated rules for selecting locations of constraints, which provide the maximum shear to the first natural frequency and to the critical force. Their minimum required number was justified.

It was proved that the installation of additional constraints (let S be the number of these constraints) could provide increasing of the value of the first natural frequency (maximum increasing to the level of $S+1$). Constraints should be placed in this connection in key points of the natural mode (buckling), corresponding to $S+1$ natural frequency (critical force) of the system without additional constraints. Special methods providing maximal value of the first natural frequency (critical force) with a minimum total stiffness of additional constraints were considered.

Another formulation of the problem of the synthesis of discrete elastic constraints providing minimum consumption of materials is presented in [7] for the problems of stability and natural oscillations.

It is necessary to select the possible location of discrete elastic constraints and to determine their stiffness so that the first natural frequency (critical load) reaches a predetermined value, the design requirements are satisfied and objective function (by weight or by volume of material of additional constraints) takes the minimum value. Special properties of natural modes (buckling modes) were defined for the systems where this problem have been realized [7].

An algorithm for solving this problem has been constructed on the basis of identified special properties of natural modes. Special properties of bars of minimum weight with restrictions on the value of the first natural frequency and critical force and varied the parameters (including various types and dimensions) of the cross-sections have been considered in [7]. Bars of different types of cross-sections were under consideration.

Let's consider these properties for bars with cross-sections in the form of an I-beam (Fig. 1) in the case of optimization where width of flange, height of section b_1, wall thickness δ_{st} and flange thickness remain unchanged. Special property for stability problem is presented below.

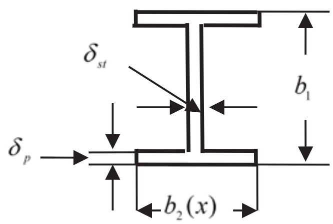

Fig. 1. Cross sections in the form of an I-beam.

Bar of I-section of minimum material consumption when the flange width is varied only (height of the cross-section, flange thickness, wall thickness are not varied) is the bar of uniform resistance with respect to reduced stresses $\overline{\sigma}_{1Pt}(x)$ in the case of buckling.

The reduced stress can be defined by formula

$$\overline{\sigma}_{1Pt}(x)=\sqrt{\sigma_{1P}^2(x)*\frac{b_1}{2*\delta_p}-\sigma_{1Pt}^2(x)*(\frac{b_1}{2*\delta_p}-1)}=const\ ; \tag{1}$$

$$(\frac{E*b_1*v_p''}{2})^2=\sigma_{1P}^2(x)\ ;\quad (\frac{b_1-2*\delta_P}{2}*E*v_p'')^2=\sigma_{1Pt}^2(x)\ , \tag{2}$$

where $\sigma_{1P}^2(x)$ is the normal stress squared in the extreme fibers of I-section; $\sigma_{1Pt}^2(x)$ is the normal stress squared in fibers at the boundary between wall and flange; E is the elastic modulus of material; v_P is the ordinate of buckling.

Vital specific property related to the considering type of cross-sections of bars of minimum weight under conditions of limitation of the value of the first natural frequency is formulated below.

Bar of I-section of minimum material consumption under condition of constant height of the cross-section, flange thickness and wall thickness and varied flange width is the bar of uniform resistance with respect to reduced stresses $\overline{\sigma}_{1\omega t}(x)$ in the case of vibration corresponding to natural mode.

The reduced stress can be defined by formula

$$\overline{\sigma}_{1\omega t}(x)=\sqrt{\sigma_{1\omega}^2(x)*\frac{b_1}{2*\delta_p}-\sigma_{1\omega t}^2(x)*(\frac{b_1}{2*\delta_p}-1)-3*E*(\omega_0)^2*v_\omega^2*\rho}=const\ ; \tag{3}$$

$$(\frac{E*b_1*v_\omega''}{2})^2=\sigma_{1\omega}^2(x)\ ;\quad (\frac{b_1-2*\delta_P}{2}*E*v_\omega'')^2=\sigma_{1\omega t}^2(x)\ , \tag{4}$$

where $\sigma_{1\omega}^2(x)$ is the normal stress squared in the extreme fibers of I-section; $\sigma_{1\omega t}^2(x)$ is the normal stress squared in fibers at the boundary between wall and flange; ω_0 is the given value of natural frequency; v_ω is the ordinate of buckling; ρ is the value of the specific gravity.

Since buckling modes are determined within constant factor reduced stresses $\overline{\sigma}_{1Pt}(x)$ and $\overline{\sigma}_{1\omega t}(x)$ can be normalized to maximum values of corresponding reduced stresses equal to one. Thus, optimal solution corresponds to reduced stresses in all cross-sections equal to one. Proximity analysis allows corresponding assessment of solution.

Properties (1) and (3) were used in [7] in order to assess the proximity of the solutions to the optimal results of the synthesis of bars of I-section of minimum consumption of materials under conditions of restrictions on the value of the critical force, or on the value of the first natural frequency. However, several possibilities of using of property (3) (particularly under conditions of restrictions on the value of the first natural frequency and necessity to take axial force into account) were not considered in [7]. It was found that property (3) can be used in the case of axial force presence.

Let's consider a bar of I-section as a vital sample in this connection (Fig. 1). The height of section b_1 is equal to 0.16 m, wall thickness δ_{st} is equal to 0.01 m, flange thickness δ_P is equal to 0.01 m. The span of this simply supported bar is equal to 6 m (Fig. 2).

Width of flange is varied within optimization process while height of section and wall thickness remain constant. In accordance with design requirements width of flange must be greater than wall thickness ($b_2(\mathrm{x})\geq\delta_{\mathrm{st}}$). Let's consider the case of loading with axial force $P=300\,\mathrm{kN}$, while beam carries a uniformly distributed mass (Fig. 1a). Corresponding discrete model (21 elements) include nodal mass equal to 21 kg. Mass of the bar was taken into account as well within optimization process. Specific gravity is equal to $\rho=7850\,\mathrm{kg/m^3}$. The given value of the first natural frequency $\omega_0\leq 21$. Random search method was used for solution, proximity analysis was based on property (3). We consider the influence of axial force on the natural frequency. Corresponding results are presented in Table 1. The first column of this table corresponds to version with constant width of flange b_2 under condition $\omega_0 = 21$. The second column contains values of the optimum width $b_2(\mathrm{x})$. Fig. 2d shows schematically flange with constant with and flange with optimal with (respectively the first and the second columns of Table 1).

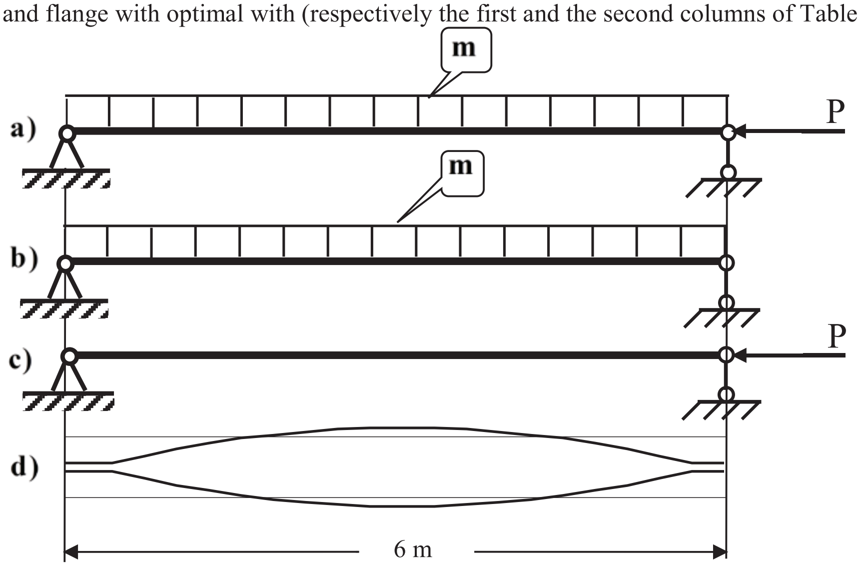

Fig. 2. Sample of simply supported bar.

Table 1. Results of analysis.

Numbers of sections	$P = 300\,\text{kN}, \omega_0 = 21$		
	b_2	$b_2(\mathrm{x})$	$\overline{\sigma}_{1\omega t}(x)$
	1	2	3
1	0.0865	0.0100	0.3643
2	0.0865	0.0128	0.9832
3	0.0865	0.0322	1.0000
4	0.0865	0.0503	0.9991
5	0.0865	0.0666	0.9952
6	0.0865	0.0802	0.9952
7	0.0865	0.0918	0.9932
8	0.0865	0.1007	0.9936
9	0.0865	0.1069	0.9950
10	0.0865	0.1107	0.9950
11	0.0865	0.1121	0.9932
12	0.0865	0.1107	0.9950
13	0.0865	0.1069	0.9950
14	0.0865	0.1007	0.9936
15	0.0865	0.0918	0.9932
16	0.0865	0.0802	0.9952
17	0.0865	0.0666	0.9952
18	0.0865	0.0503	0.9991

19	0.0865	0.0322	1.0000
20	0.0865	0.0128	0.9832
21	0.0865	0.01000	0.3643

The third column of Table 1 shows the values of reduced stress $\overline{\sigma}_{1\omega t}(x)$ defined by formula (3). Difference between value of reduced stress and one allows optimality assessment of solution. The value of $\overline{\sigma}_{1\omega t}(x)$ is close to 1 for all cross-sections (except boundary ones). Design requirements are satisfied at boundary sections ($b_2(\mathrm{x}) = \delta_{\mathrm{st}}$). However, properties (1) and (3) are not met. The distinctive sample confirms that the property (3) is satisfied, taking into account the effect of a axial force on the natural frequency. The objective function (volume of material) decreased as a result of optimization from 0.018775 m^3 to 0.016610 m^3 (i.e. it is down by 13.04 percent).

The ratio between the axial force and frequency may vary while in operation. The literature often argues that the optimal systems become less reliable after change in the operation conditions. Thus, it is advisable to set the range of the frequency and the axial forces satisfying stability conditions and requirements for the first natural frequency. We can use so-called theory of boundary surface construction, proposed by P.F. Papkovich. Detailed method of determination of stability domain under the action of axial force with allowance for effect of axial force on the natural frequencies spectrum is considered in [13]. This stability domain is constructed in the coordinate system $\omega^2 - P$ [13]. We can build boundary line (separating stability domain) point by point (based on three points).

Let's consider optimal bar with dimensions given in column 2 of Table 1 in this connection. One of the points of this line is determined by values $\omega = 21$, $\omega^2 = 441$ and $P = 300\,\mathrm{kN}$, which were used within optimization process (point a, Fig. 3). The point b corresponds to the first natural frequency $\omega = 28.1616$, $\omega^2 = 793.0757146$, $P = 0$ of optimal bar without the influence of axial force (Fig. 2b). The point c corresponds to the critical force, excluding the effect of frequency $P = 674.9747\,\mathrm{kN}, \omega_0 = 0$ (Fig. 2c). Arbitrary combinations of values of the natural frequencies and axial forces within the domain bounded by the line $(b-a-c)$ and coordinate axes will meet stability conditions and frequency limitation.

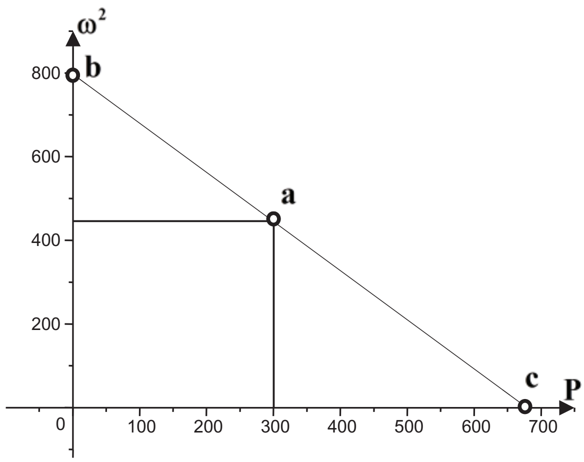

Fig. 3. Sample of stability domain.

In order to compare the domain of safe combinations of natural frequency and the axial force for optimal bar with the equivalent domain for bar with a constant width of wall we should plot boundary line point by point (based on three points). Corresponding width of flange of this bar is given in column 1 of Table 1. We used $\omega = 21$, $\omega^2 = 441$ and $P = 300\,\mathrm{kN}$ (i.e. point a) for its definition. The first natural frequency of the bar without the influence of axial force is equal to $\omega = 28.1293$, $\omega^2 = 791.2965$ ($P = 0$). Point b is a corresponding point with an accuracy up to the fraction of percent. Corresponding critical force is equal to $P = 678.2725\,\text{kN}$ (without the influence of frequency, $\omega_0 = 0$). Point c is a corresponding point with an accuracy up to the fraction of percent. Thus, domains of safe combinations of optimal bar and bar with a constant width of wall are practically the same, but the material consumption of optimal bar is less then material consumption of bar with a constant width by 13.04%.

There is a formulation of the optimization problem, eliminating concerns that the optimal system may not have an adequate supply. This formulation is presented below.

So-called *domain of possible actions* is constructed in the space of actions. Each point determines the *originating system* in the space of varied parameters. *Domain of permissible actions* is constructed in the space of actions for this system. All combinations of actions within domain of permissible actions don't break restrictions of originating system. *If the domain of possible actions lies within domain of permissible actions, the originating system is permissible (acceptable). Optimal system, which is selected from all permissible systems, must give extreme value to objective function (functional), for instance for systems with minimum material consumption.*

Correct application of this formulation provide necessary safety factor for optimal system. As is known conventional design lead to excessive safety factors for structural members (for instance, in several cross-sections of the bar).

Analysis of the theory of optimal design (taking into account the development of modern computer technologies and sophisticated software) allows investigator to overcome various problems, which normally limited the widespread use of optimization techniques in design practice. Special properties of optimal systems, various theories of strength, wide range of loadings (actions) and constraints, different types of systems and structural members (thin-walled structures, compound bars, shell structures, optimal pre-stressing, random actions, design (structural) constraints and so on) should be under consideration.

It is also advisable to widen using of the special properties of optimal systems to assess the difference between solutions, obtained by various methods, and corresponding optimal solution.

Moreover it is necessary to continue development methods of analysis and design of optimal systems based on the use of their special properties. The main aim is development of methods for synthesis of optimal structures or, in other words, approaches to design of structures with preset properties.

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